Nonperturbative Spontaneous Symmetry Breaking

Vladimir Dzhunushaliev *

August 14, 2002

Institut für Theoretische Physik, Freie Universität Berlin, Arnimallee 14, D-14195, Berlin, Germany
and
Dept. Phys. and Microel. Engineer., Kyrgyz-Russian Slavic University Bishkek, Kievskaya Str. 44, 720000, Kyrgyz Republic

Abstract

A mechanism for nonperturbative spontaneous symmetry breaking is proposed. It based on some properties of *interacting* field operators. As the consequences an additional terms like to m^2A^2 appears in the initial Lagrangian.

1 Introduction

One of the most astonishing results of quantum field theory is spontaneous symmetry breaking. One can say that it is the situation when something arises from the quantization. This means that on the quantum level one has something that was not present on the classical level. Coleman and Weinberg [1] write : " ... higher-order effects may qualitatively change the character of a physical theory ... ". The main goal of the Coleman-Weinberg mechanism is to derive a Higgs potential from more fundamental principles, with as few arbitrary parameters as possible. In this mechanism the Higgs potential in induced by radiative corrections, rather than being inserted by hand. In this approach one can summarize over higher-loop graphs to induce an effective potential, which may then produce spontaneous symmetry breaking. Of coarse it can be only in the theories with interactions (where we have these higher-loop graphs) : *i.e.* a nonlinearity (in Lagrangian) can lead to the interesting consequences for the quantized theory. It is not very surprisingly because on the classical level we have the same : very simple behaviour of a classical linear theory can be changed on the very complicated and surprising behaviour of a classical nonlinear theory.

*E-mail: dzhun@hotmail.kg

For example, in nonlinear theories we have monopoles, instantons, black holes, strange attractors and so on. On the quantum level we can expect once more amasing stuffs if we add some nonlinear terms in Lagrangian. Probably one of such manifestations of a nonlinearity is confinement in the QCD.

The problem here is that we do not have detailed techniques for the nonperturbative calculations. Even on the perturbative level we do not sure that the result after the sum over all Feynman diagrams will the same as after the sum over a finite number of Feynman diagrams. Nevertheless according to perturbative calculations we know that they change an initial Lagrangian so that an extra potential term arises in the Lagrangian (Coleman-Weinberg mechanism).

In this paper we will try to work out a nonpertutbative method which can be applied for strongly interacting fields and to show that extra terms (or nonperturbative spontaneous symmetry breaking) appear in an initial Lagrangian if the product of field operators have some properties concerning to the rearrangement of the brackets in a nonlinear potential term.

2 Nonlinear term in non-Abelian gauge theories

Our basic attention here is devoted to the non-Abelian gauge field SU(2) (for the simplicity we will consider only this gauge group). The Lagrangian is

$$\mathcal{L} = \frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} \tag{1}$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu$ is the field strength and A^a_μ is the gauge potential; g is the coupling constant; ϵ^{abc} are the structural constants of the gauge group SU(2); $a = 1, 2, 3$. In the quantum case we have the operators $\hat{A}^a_\mu$ and $\hat{F}^a_{\mu\nu} = \partial_\mu \hat{A}^a_\nu - \partial_\nu \hat{A}^a_\mu + g\epsilon^{abc} \hat{A}^b_\mu \hat{A}^c_\nu$. Let us underline that all operators considered here are the operators of *interacting* fields in contrast with the perturbative techniques where these operators describe *non-interacting* fields.

Let us consider the nonlinear part of the field strength operator $\hat{F}^a_{\mu\nu}$: $(\hat{F}_{nl})^a_{\mu\nu} = \epsilon^{abc} \hat{A}^b_\mu \hat{A}^c_\nu$. At first we assume that this product do not have any singularity as it is the product of *interacting* fields. Physically it means that there are situations in interacting field theories where these singularities do not occur (*e.g.* for flux tubes in Abelian or non-Abelian theory quantities like the "electric" field inside the tube, $\langle E^a_z \rangle < \infty$, and energy density $\varepsilon(x) = \langle (E^a_z)^2 \rangle < \infty$ are nonsingular). Here we take as an assumption that such singularities do not occur.

Thus we have such nonlinear term

$$\begin{aligned}
\left(\left(\hat{F}_{nl} \right)^a_{\mu\nu} \right) \left(\left(\hat{F}_{nl} \right)^{a\mu\nu} \right) = \epsilon^{abc} \epsilon^{ade} \left(\hat{A}^b_\mu \hat{A}^c_\nu \right) \left(\hat{A}^{d\mu} \hat{A}^{e\nu} \right) &= \\
\left(\delta^{bd}\delta^{ce} - \delta^{be}\delta^{cd} \right) \left(\hat{A}^b_\mu \hat{A}^c_\nu \right) \left(\hat{A}^{d\mu} \hat{A}^{e\nu} \right) &= \\
\left(\hat{A}^b_\mu \hat{A}^c_\nu \right) \left(\hat{A}^{b\mu} \hat{A}^{c\nu} \right) - \left(\hat{A}^b_\mu \hat{A}^c_\nu \right) \left(\hat{A}^{c\mu} \hat{A}^{b\nu} \right). &
\end{aligned} \tag{2}$$

Our main assumption is that this nonlinear expression has *some properties* that allows us to say that

$$\left(\hat{A}^a_\mu \hat{A}^b_\nu\right)\left(\hat{A}^{a\mu}\hat{A}^{b\nu}\right) - \left(\hat{A}^a_\mu \hat{A}^{a\mu}\right)\left(\hat{A}^b_\nu \hat{A}^{b\nu}\right) \neq 0 \tag{3}$$

and the difference is not connected with the commutator $[\hat{A}^a_\mu, \hat{A}^b_\nu]$. Physically it means that the square of nonlinear part $(F_{nl})^a_{\mu\nu}$ of the field strength is not equal to the square of the vector square $\hat{A}^4 = (\hat{A}^a_\mu \hat{A}^{a\mu})^2$, *i.e.* the r.h.s. of Eq. (3) is nonzero. In some sense it is like to Cooper pairing in the superconductivity. The initial potential term for electrons is $\left(\hat{\psi}^+_\beta \left(\hat{\psi}^+_\alpha \hat{\psi}_\alpha\right)\hat{\psi}_\beta\right)$ and after some manipulations we have $\left(\hat{\psi}_\alpha \hat{\psi}_\beta\right)\left(\hat{\psi}^+_\gamma \hat{\psi}^+_\delta\right)$ wher every pair of brackets $(\cdots)(\cdots)$ has an independent physical meaning : each pair describes annihilation and creation of Cooper pair. We note that these pairs were not present in the initial Lagrangian. Another words at first we had electrons and then Cooper pairs.

The difference (3) is connected only with the nonlinearity of the term $(\hat{F}_{nl})^2$. In order to calculate this difference we will consider at first very simple example.

3 Simple example

Let $\hat{a}$ and $\hat{b}$ are operators which commutates

$$\hat{a}\hat{b} - \hat{b}\hat{a} = 0 \tag{4}$$

but have a nonassociative property

$$\left(\hat{a}\hat{b}\right)\left(\hat{a}\hat{b}\right) - (\hat{a}\hat{a})\left(\hat{b}\hat{b}\right) \neq 0. \tag{5}$$

For the simplicity we assume that

$$\left(\hat{a}\hat{b}\right)\hat{b} = \hat{a}\left(\hat{b}^2\right) = \hat{a}\hat{b}^2 \tag{6}$$

$$\hat{a}\left(\hat{a}\hat{b}\right) = \left(\hat{a}^2\hat{b}\right) = \hat{a}^2\hat{b} \tag{7}$$

It means that the algebra of these operators is an alternative algebra and

$$\left(\hat{a}\hat{b}\right)\hat{a} = \hat{a}\left(\hat{b}\hat{a}\right). \tag{8}$$

Then Eq. (5) can be written as follows

$$\left(\hat{a}\hat{b}\right)\left(\hat{a}\hat{b}\right) - (\hat{a}^2)\left(\hat{b}^2\right) \neq 0. \tag{9}$$

In order to calculate this commutations relationship we compare its with the ordinary commutators in a linear field theory

$$\hat{\phi}(x)\hat{\phi}(y) - \hat{\phi}(y)\hat{\phi}(x) = -i\hbar D(x-y) \tag{10}$$

where $\hat{\phi}(x)$ is some field operator and $D(x-y)$ is some function. We see that at the l.h.s. we have the production of two operators and at the r.h.s. the number of operators is $(2-2)=0$.

This procedure we would like to apply for the expression (9). At the l.h.s. we have the production of four operators, consequently at the r.h.s. we should have the production of $(4-2)=2$ operators. Thus

$$\left(\hat{a}\hat{b}\right)\left(\hat{a}\hat{b}\right)-\left(\hat{a}^2\right)\left(\hat{b}^2\right)=\lambda\left(\hat{a}-\hat{b}\right)\left(\hat{a}-\hat{b}\right)=\lambda\left(\hat{a}-\hat{b}\right)^2. \tag{11}$$

The r.h.s. of this equation is choosed by such manner that it have to be zero in the case $\hat{a}=\hat{b}$ and our definition (11) satisfies to this requirement.

Another approach to this formula is the following. According Eq.'s (6) (7) we have

$$\left(\hat{a}^2\right)\left(\hat{b}^2\right)=\left(\hat{a}^2\hat{b}\right)\hat{b}=\left(\hat{a}\left(\hat{a}\hat{b}\right)\right)\hat{b}=\left(\hat{a}\left(\hat{b}\hat{a}\right)\right)\hat{b}=\left(\left(\hat{a}\hat{b}\right)\hat{a}\right)\hat{b}=$$
$$\left(\hat{a}\hat{b}\right)\left(\hat{a}\hat{b}\right)-\lambda\left(\hat{a}-\hat{b}\right)^2. \tag{12}$$

Thus, $-\lambda(\hat{a}-\hat{b})^2$ is an associator :

$$\left(\left(\hat{a}\hat{b}\right)\hat{a}\right)\hat{b}-\left(\hat{a}\hat{b}\right)\left(\hat{a}\hat{b}\right)=-\lambda\left(\hat{a}-\hat{b}\right)^2. \tag{13}$$

We would like to underline again that the differences (11)-(13) *are not connected with the commutator* $[\hat{a},\hat{b}]$ which can be zero. This is only a manifestation of the nonlinearity of the corresponding expression $(\hat{a}\hat{b})(\hat{a}\hat{b})$ or $(\hat{a}^2)(\hat{b}^2)$.

4 Non-Abelian case

Let us come back to more realistic case : non-Abelian gauge theories. Bear in mind Eq.(12) we write the following

$$\begin{aligned}
&\left(\hat{A}^a_\mu\hat{A}^b_\nu\right)\left(\hat{A}^{a\mu}\hat{A}^{b\nu}\right)=\\
&\left(\left(\hat{A}^a_\mu\hat{A}^b_\nu\right)\hat{A}^{a\mu}\right)\hat{A}^{b\nu}+\alpha\hat{A}^a_\mu\hat{A}^{a\mu}-2\alpha^{ab\mu\nu}\hat{A}^a_\mu\hat{A}^b_\nu+\hat{A}^b_\nu\hat{A}^{b\nu}=\\
&\left(\hat{A}^a_\mu\left(\hat{A}^b_\nu\hat{A}^{a\mu}\right)\right)\hat{A}^{b\nu}+(\cdots)=\left(\hat{A}^a_\mu\left(\hat{A}^{a\mu}\hat{A}^b_\nu\right)\right)\hat{A}^{b\nu}+(\cdots)=\\
&\left(\left(\hat{A}^a_\mu\hat{A}^{a\mu}\right)\hat{A}^b_\nu\right)\hat{A}^{b\nu}+(\cdots)=\left(\hat{A}^a_\mu\hat{A}^{a\mu}\right)\left(\hat{A}^b_\nu\hat{A}^{b\nu}\right)+(\cdots)
\end{aligned} \tag{14}$$

α and $\alpha^{ab\mu\nu}$ are some numerical coefficients with the following relation

$$\alpha=\alpha^{aa\mu\mu} \tag{15}$$

here we do not summarise over a and μ. This constrain follows from the fact that l.h.s. of Eq.(14) must coincide with the first term on the r.h.s. of this equation in the case when $a = b$ and $\mu = \nu$. In fact the second term at the r.h.s. of Eq.(14) is like to the r.h.s. of Eq.(11). The same is for the next term in Eq. (2)

$$\begin{aligned}
&\left(\hat{A}^a_\mu \hat{A}^b_\nu\right)\left(\hat{A}^{b\mu}\hat{A}^{a\nu}\right) = \\
&\left(\left(\hat{A}^a_\mu \hat{A}^b_\nu\right)\hat{A}^{b\mu}\right)\hat{A}^{a\nu} + \beta\hat{A}^a_\mu\hat{A}^{a\mu} - 2\beta^{ab\mu\nu}\hat{A}^a_\mu\hat{A}^b_\nu + \beta A^b_\nu\hat{A}^{b\nu} = \\
&\left(\hat{A}^a_\mu\left(\hat{A}^b_\nu\hat{A}^{b\mu}\right)\right)\hat{A}^{a\nu} + (\cdots) = \left(\hat{A}^a_\mu\left(\hat{A}^{b\mu}\hat{A}^b_\nu\right)\right)\hat{A}^{a\nu} + (\cdots) = \\
&\left(\left(\hat{A}^a_\mu\hat{A}^{b\mu}\right)\hat{A}^b_\nu\right)\hat{A}^{a\nu} + (\cdots) = \left(\hat{A}^a_\mu\hat{A}^{b\mu}\right)\left(\hat{A}^b_\nu\hat{A}^{a\nu}\right) + (\cdots)
\end{aligned} \tag{16}$$

where β and $\beta^{ab\mu\nu}$ are some numerical coefficients with the same relation between these coefficients

$$\beta = \beta^{aa\mu\mu} \tag{17}$$

here we do not summarise over a and μ. Here we assume that

$$\left(\hat{A}^a_\alpha(x)\hat{A}^b_\beta(x)\right)\hat{A}^c_\gamma(x) = \hat{A}^a_\alpha(x)\left(\hat{A}^b_\beta(x)\hat{A}^c_\gamma(x)\right) = \hat{A}^a_\alpha(x)\hat{A}^b_\beta(x)\hat{A}^c_\gamma(x) \tag{18}$$

and the commutator of the *interacting* fields *at the same point* is zero

$$\left[\hat{A}^a_\alpha(x), \hat{A}^b_\beta(x)\right] = 0. \tag{19}$$

It is necessary to note that all these calculations (14)-(19) was done for the fields in one point (x).

It is evidently that our extra terms $\left(\alpha\hat{A}^a_\mu\hat{A}^{a\mu} - 2\alpha^{ab\mu\nu}\hat{A}^a_\mu\hat{A}^b_\nu + \hat{A}^b_\nu\hat{A}^{b\nu}\right)$ and $\left(\beta\hat{A}^a_\mu\hat{A}^{a\mu} - 2\beta^{ab\mu\nu}\hat{A}^a_\mu\hat{A}^b_\nu + \beta A^b_\nu\hat{A}^{b\nu}\right)$ break an initial symmetry of the given Lagrangian and consequently it is similar to Coleman-Weiberg symmetry breaking but on the nonperturbative level.

5 Some explanations

Now we would like to explain why we need with equations (14)-(16). Let us assume that we have a quantum state $|Q\rangle$ and the action of the field operator $\hat{A}^a_\mu$ on this state is $|\Phi\rangle = \hat{A}^a_\mu|Q\rangle$. For the nonlinear term the problem is that we do not know the action of this term by the direct way

$$\left(\hat{A}^a_\mu\hat{A}^b_\nu\right)\left(\hat{A}^{a\mu}\hat{A}^{b\nu}\right)|Q\rangle = ? \tag{20}$$

because we do not know the action of the operator $(\hat{A}^{a\mu}\hat{A}^{b\nu})$ on the quantum state $|Q\rangle$. In order to determine the action (20) we need for the following transformations

$$\begin{aligned}
&\left(\hat{A}^a_\mu\hat{A}^b_\nu\right)\left(\hat{A}^{a\mu}\hat{A}^{b\nu}\right)|Q\rangle \to \left(\left(\hat{A}^a_\mu\hat{A}^b_\nu\right)\hat{A}^{a\mu}\right)\left(\hat{A}^{b\nu}|Q\rangle\right) = \\
&\left(\hat{A}^a_\mu\hat{A}^b_\nu\right)\left(\hat{A}^{a\mu}|\Phi_1\rangle\right) = \hat{A}^a_\mu\left(\hat{A}^b_\nu|\Phi_2\rangle\right) = \hat{A}^a_\mu|\Phi_3\rangle = |\Phi_4\rangle
\end{aligned} \tag{21}$$

where $|\Phi_1\rangle = \hat{A}^a_\mu|Q\rangle$ and so on for every $|\Phi_i\rangle$, $i = 2, 3, 4$.

6 Discussions and conclusions

In this paper we have shown that a nonlinear potential in quantum non-Abelian gauge theory can lead to the appearance of some extra terms in Lagrangian. In Ref. [2] it was shown that Ginzburg - Landau equation can be derived from a pure non-Abelian gauge theory. There was assumed that the quantization of such theory leads to the appearance of m^2A^2-like term in the initial Lagrangian. In this paper we suggested a nonperturbative mechanism for this phenomenon.

It is interesting to apply proposed here mechanism to the $\lambda\phi^4-$theory (the simplest quantum field theory with the nonlinearity). But there we have to be careful as we consider the simplest case with (6)-(8) limitations. These limitations do not allow us to derive anything interesting for the $\lambda\phi^4-$theory since the presented here mechanism is not working because with our (6)-(8) restrictions we have $(\phi^2)(\phi^2) = \phi^4$ and there are not any distinction between these two brackets with ϕ^2. Consequently for more realistic case we should consider the theory without (6)-(8) constrains.

In Ref. [3] have been investigated a possibility that the operators of quantum fields with a nonlinear potential can have nonassociative properties. In this paper we continue this approach but there is one very essential disctinction : we assume that the remainder in Eq. (3) is not a numerical function but is an operator which is the product of fields operators.

In Ref's [4] it is shown that a Meissner-like effect in non-Abelian gauge theories arises. There was applied a nonperturbative quantization techniques based on the Heisenberg's approach to a nonlinear spinor field. Heisenberg's idea is to use field operator equations for receiving an infinite equations set for all Green's functions. In this paper (remaining in the frame of Heisenberg's approach) we show that this nonperturbative mechanism leads to spontaneously symmetry breaking.

Finally we whould like to say that probably nonperturbative quantum field theory with a strong interaction will have very unusual and unexpected properties in contrast with quantum field theory with a small coupling constant. This difference is similar with the difference between analytical and differentiable (but nonanalytical) functions. The first case is similar to a quantum field theory with a small coupling constant, where we can expand the function in Taylor-series which for the quantum theory is Feynman diagrams. In the second case the function can not be expanded in any Taylor-series and respectively Feynman techniques can not be applied for such kind of quantum field theories. Probably the QCD belong to the last case that leads to the fact that confinement can not be explained on the language of Feynman diagrams. Our opinion is that in this case we can use the Heisenberg's nonperturbative quantization method [5]. Something like this takes place in the classical case : the nonlinear classical theories have such nonperturbative phenomena as : self-ogranisation, monopoles, instantons, black holes, strange attractors and so on which are absent in linear theories and they can not be derived by the perturbative way.

7 Acknowledgments

I am very grateful for the DAAD for the financial support and Prof. Hagen Kleinert for the hospitality in his research group. Partially this work was supported by Alexander von Humboldt Foundation.

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